



VERROU: panorama of localization method

CERN: Compute & Accelerator Forum
02/12/25

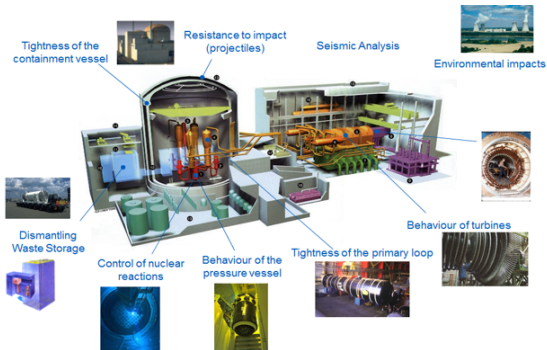
Bruno Lathuilière (EDF R&D)

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ANR-20-CE46-0009.



Numerical simulation at EDF

- ◆ Guarantee safety
- ◆ Improve performances/costs
- ◆ Ageing issues



Numerical software at EDF

Large number of in-house codes:

- ◆ code_aster (Thermo-mechanic)
- ◆ code_saturn (CFD)
- ◆ open_telemac (free surface flow)
- ◆ salome (Simulation platform)
- ◆ ...

Code properties:

- ◆ Huge code base
- ◆ Various languages (C/C++, Fortran, Python ...)
- ◆ External libraries (MUMPS, numpy ...)
- ◆ V&V

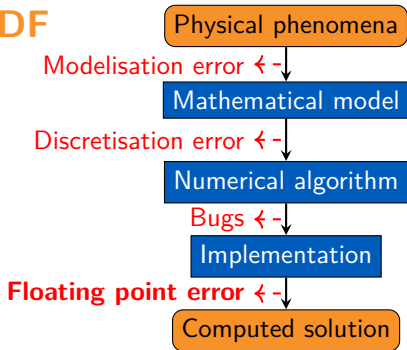
Numerical software at EDF

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- V&V**



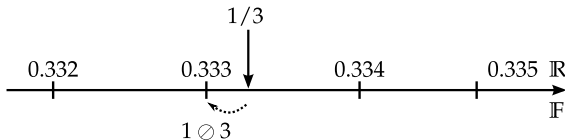
VERROU development:

- Binary instrumentation based on valgrind
- Asynchronous stochastic arithmetic
- Error estimation and error localization**

Floating point error

◆ Floating point representation with limited precision

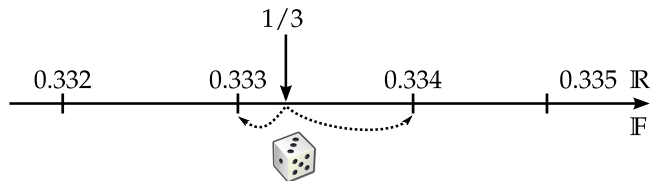
- ▶ [float] binary, 24 significand bits ($\simeq 10^{-7}$)
- ▶ [double] binary, 53 significand bits ($\simeq 10^{-16}$)
- ▶ [pedagogic example] decimal, 3 significand digits (% - %%)



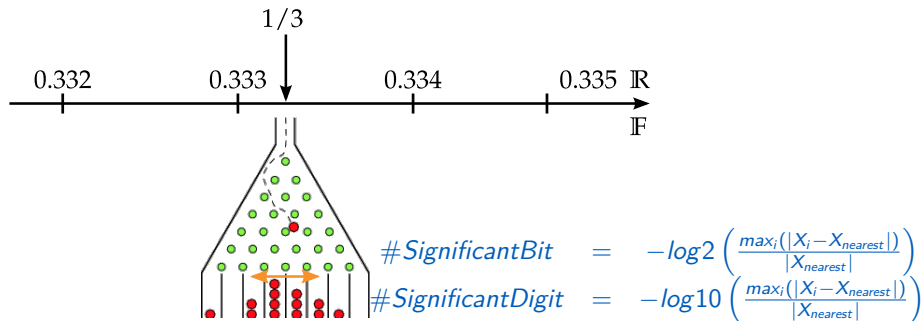
◆ Floating point computation $\neq$ Real computation

- ▶ rounding error $a \oplus b \neq a + b$
- ▶ associativity loss $(a \oplus b) \oplus c \neq a \oplus (b \oplus c)$

Stochastic arithmetic for numerical verification



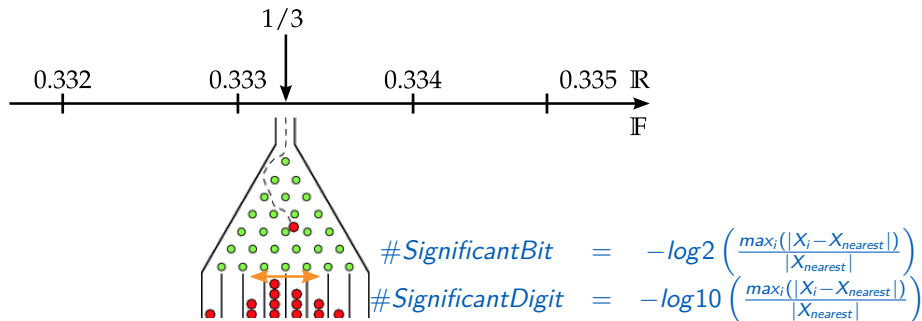
Stochastic arithmetic for numerical verification



Instruction	Eval. Nearest	Eval. 1	Eval. 2	Eval. 3
$a = 1/3$	0.333	$0.333_{\downarrow}$	$0.334_{\uparrow}$	$0.334_{\uparrow}$
$b = a \times 3$	0.999	0.999	$1.00_{\downarrow}$	$1.01_{\uparrow}$

$\#SignificantDigit \approx 1.95$

Stochastic arithmetic for numerical verification



Instruction	Eval. Nearest	Eval. 1	Eval. 2	Eval. 3
$a = 1/3$	0.333	$0.333_{\downarrow}$	$0.334_{\uparrow}$	$0.334_{\uparrow}$
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$\#SignificantDigit \approx 1.95$

- Easily compatible with binary instrumentation (**Verrou** based on valgrind) or low-level LLVM pass (Verificarlo)
- Few false positive detection (due to asynchronous approach and dedicated stochastic rounding mode)

Example : rectangle method

$$\int_a^b f(x)dx \approx \sum_i^n \frac{(b-a)}{n} F(X(i)) \quad \text{avec} \quad X(i) = a + (i + 0.5) \frac{(b-a)}{n}$$

Listing – integrate.hxx

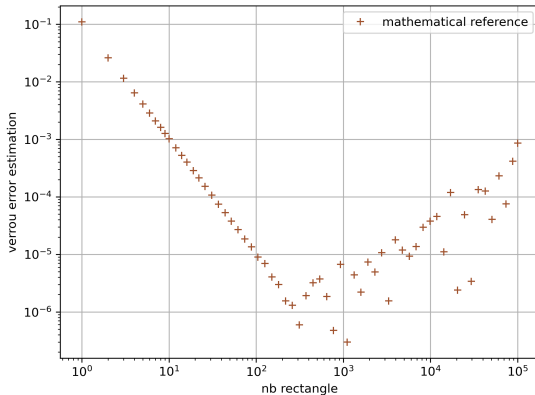
```
8 template <typename T, class REALTYPE>
9 REALTYPE integrate (const T& f, REALTYPE a, REALTYPE b, unsigned int n) {
10     // Integration step
11     const REALTYPE dx = (b - a) / n;
12     // Naive integration Loop
13     REALTYPE sum = 0.;
14     for (REALTYPE x = a + (REALTYPE)0.5 * dx ;
15         x < b ;
16         x += dx) {
17         sum += dx * f(x);
18     }
19     return sum;
20 }
```

Listing – unitTest.cxx (Verification: $\int_0^{\frac{\pi}{2}} \cos(x)dx = 1$)

```
22 void testConvergence (const float & step) {
23     std::cout << std::scientific << std::setprecision(17);
24     const size_t maxN= 100000;
25     for (unsigned int n = 1 ; n <= maxN ;
26         n = std::max (std::min(maxN, (size_t)(step*n)), n+1)) {
27
28         float res = integrate (std::cos,0., M_PI_2, n);
29
30         float err = std::abs(1 - res);
31
32         // 3 columns output: Nrectangles Result Error
33         std::cout << std::setw(10) << n << " " << res << " " << err << std::endl;
34     }
35 }
```

Example: convergence analysis

1	1.11072075366973877e+00	1.10720753669738770e-01
10	1.00102877616882324e+00	1.02877616882324219e-03
100	1.00001001358032227e+00	1.00135803222656250e-05
1000	9.99992489814758301e-01	7.51018524169921875e-06
10000	1.00000333786010742e+00	3.33786010742187500e-06
100000	9.99139010906219482e-01	8.60989093780517578e-04



Example: manual analysis

```
$ ./unitTest > unittest.output.txt
```

```
unittest.output.txt
```

```
1 1.11072075366973877e+00 1.10720753669738770e-01  
10 1.00102877616882324e+00 1.02877616882324219e-03  
100 1.00001001358032227e+00 1.00135803222656250e-05  
1000 9.99992489814758301e-01 7.51018524169921875e-06  
10000 1.00000333786010742e+00 3.33786010742187500e-06  
100000 9.99139010906219482e-01 8.60989093780517578e-04
```

Example: manual analysis

```
$ ./unitTest > unittest.output.txt
```

```
$ valgrind --tool=verrou --rounding-mode=random --libm=instrumented ./unitTest >  
unittest.output-random_1.txt
```

unittest.output.txt

```
1 1.11072075366973877e+00 1.10720753669738770e-01  
10 1.00102877616882324e+00 1.02877616882324219e-03  
100 1.00001001358032227e+00 1.00135803222656250e-05  
1000 9.99992489814758301e-01 7.51018524169921875e-06  
10000 1.00000333786010742e+00 3.33786010742187500e-06  
100000 9.99139010906219482e-01 8.60989093780517578e-04
```

unittest.output_random_1.txt

```
1 1.11072075366973877e+00 1.10720753669738770e-01  
10 1.00102853775024414e+00 1.02853775024414062e-03  
100 1.00001108646392822e+00 1.10864639282226562e-05  
1000 1.00000536441802979e+00 5.36441802978515625e-06  
10000 1.00001549720764160e+00 1.54972076416015625e-05  
100000 1.00013923645019531e+00 1.39236450195312500e-04
```

Example: manual analysis

```
$ ./unitTest > unittest.output.txt
```

```
$ valgrind --tool=verrou --rounding-mode=random --libm=instrumented ./unitTest >  
unittest.output-random_1.txt
```

```
$ meld unittest.output.txt unittest.output-random_1.txt
```

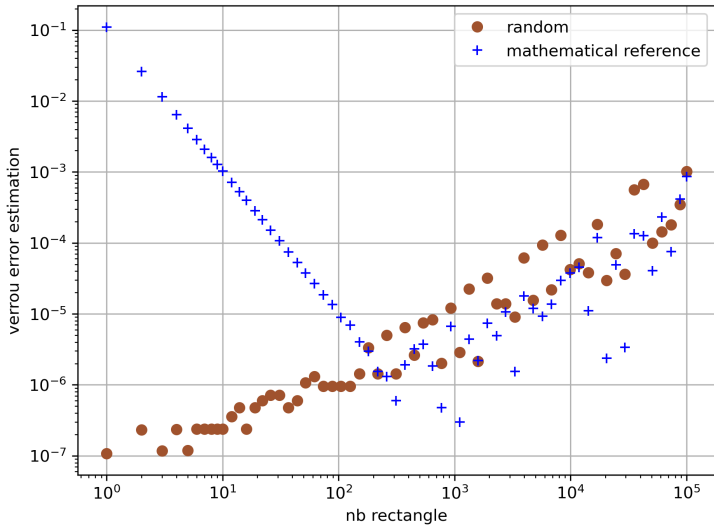
unittest.output.txt

```
1 1.11072075366973877e+00 1.10720753669738770e-01  
10 1.00102877616882324e+00 1.02877616882324219e-03  
100 1.00001001358032227e+00 1.00135803222656250e-05  
1000 9.99992489814758301e-01 7.51018524169921875e-06  
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```

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1 1.11072075366973877e+00 1.10720753669738770e-01  
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10000 1.00001549720764160e+00 1.54972076416015625e-05  
100000 1.00013923645019531e+00 1.39236450195312500e-04
```

Error plot with 20 samples



Delta-debug: *trial and error* search algorithm

Delta-debug search:

```
1 verrou_dd_line --nruns=5 ddRun.sh ddCmp.py  
                                cmpScript: ddCmp.py
```

runScript: ddRun.sh

```
1 #!/bin/bash  
2 OUTDIR=$1  
3 valgrind --tool=verrou --rounding-mode=random  
  --libm=instrumented\  
4 ./unitTest >${OUTDIR}/res.dat
```

```
6 def extract(rep):  
7     lines=(open(os.path.join(rep, "  
    "res.dat"))).readlines()  
8     return re.split(" ", lines[-1].strip())[1]  
9  
10 if __name__=="__main__":  
11     if len(sys.argv)==2: #extract for  
        verrou_plot_stat  
12         print(extract(sys.argv[1]))  
13     if len(sys.argv)==3: #cmp for verrou_dd_  
        refValue=float(extract(sys.argv[1]))  
14         value=float(extract(sys.argv[2]))  
15         relDist=abs((value-refValue)/refValue)  
16         if relDist < 1e-5: sys.exit(0)  
17         else: sys.exit(1)
```

ddmin	filename:line	demangled symbol name
ddmin0	integrate.hxx:17	testConvergence(float const &)
ddmin1	integrate.hxx:16	testConvergence(float const &)

Valgrind developer point of view:

- ◆ need to generate a search space: list of symbols (or line if compiled with -g) containing floating point operations;
- ◆ need to run a specific configuration (set instrumented /not instrumented)

Code coverage for unstable branches detection

nearest coverage		random coverage	
-	:8:template <typename T, class REALTYPE>	-	:8:template <typename T, class REALTYPE>
6	:9:REALTYPE integrate (const T& f, REALTYPE a, REALTYPE b, unsigned int n) {	6	:9:REALTYPE integrate (const T& f, REALTYPE a, REALTYPE b, unsigned int n) {
-	:10: // Integration step	-	:10: // Integration step
6	:11: const REALTYPE dx = (b - a) / n;	6	:11: const REALTYPE dx = (b - a) / n;
-	:12: // Naive integration Loop	-	:12: // Naive integration Loop
6	:13: REALTYPE sum = 0.;	6	:13: REALTYPE sum = 0.;
6	:14: for (REALTYPE x = a + (REALTYPE)0.5 * dx ;	6	:14: for (REALTYPE x = a + (REALTYPE)0.5 * dx ;
111000	:15: x < b ;	111179	:15: x < b ;
110994	:16: x += dx) {	111173	:16: x += dx) {
110994	:17: sum += dx * f(x);	111173	:17: sum += dx * f(x);
-	:18: }	-	:18: }
-	:19: return sum;	-	:19: return sum;
-	:20:}	-	:20:}

- ◆ Need to recompile with coverage option (-fcoverage).
- ◆ Need to rerun delta-debug.
- ◆ Need time to interpret results due to false-positive.

BasicBloc coverage

ddmin1 (integrate.hxx:16) random		nearest	
1	:②iostream(74)unitTest.cxx(65)	1	:②iostream(74)unitTest.cxx(65)
1	:②unitTest.cxx(65)iostream(74)	1	:②unitTest.cxx(65)iostream(74)
111168	:②cmath(185)	110988	:②cmath(185)
6	:②integrate.hxx(15)cmath(185)	6	:②integrate.hxx(15)cmath(185)
111174	:②integrate.hxx(15-17) F?	110994	:②integrate.hxx(15-17) F?
4	:②iomanip(240)integrate.hxx(11,13-15) F?	4	:②iomanip(240)integrate.hxx(11,13-15) F?
1	:②iomanip(240)stl_algobase.h(237) ← integrate.hxx(11,13-15) F? ostream(196) F ios_base.h(84,88,731)integrate.hxx(11,13-15) F?	1	:②iomanip(240)stl_algobase.h(237) ← integrate.hxx(11,13-15) F? ostream(196) F ios_base.h(84,88,731)integrate.hxx(11,13-15) F?
≈23 lines skipped ≈			

ddmin0 (integrate.hxx:17) random		nearest	
1	:②iostream(74)unitTest.cxx(65)	1	:②iostream(74)unitTest.cxx(65)
1	:②unitTest.cxx(65)iostream(74)	1	:②unitTest.cxx(65)iostream(74)
110988	:②cmath(185)	110988	:②cmath(185)
6	:②integrate.hxx(15)cmath(185)	6	:②integrate.hxx(15)cmath(185)
110994	:②integrate.hxx(15-17) F?	110994	:②integrate.hxx(15-17) F?
4	:②iomanip(240)integrate.hxx(11,13-15) F?	4	:②iomanip(240)integrate.hxx(11,13-15) F?
1	:②iomanip(240)stl_algobase.h(237) ← integrate.hxx(11,13-15) F? ostream(196) F ios_base.h(84,88,731)integrate.hxx(11,13-15) F?	1	:②iomanip(240)stl_algobase.h(237) ← integrate.hxx(11,13-15) F? ostream(196) F ios_base.h(84,88,731)integrate.hxx(11,13-15) F?
≈23 lines skipped ≈			

◆ Can be generated automatically with `post_verrou_dd`

Example : come back to code

Listing – integrate.hxx

```
8 template <typename T, class REALTYPE>
9 REALTYPE integrate (const T& f, REALTYPE a, REALTYPE b, unsigned int n) {
10     // Integration step
11     const REALTYPE dx = (b - a) / n;
12     // Naive integration Loop
13     REALTYPE sum = 0.;
14     for (REALTYPE x = a + (REALTYPE)0.5 * dx ;
15         x < b ;
16         x += dx) {
17         sum += dx * f(x);
18     }
19     return sum;
20 }
```

- ◆ Convert the floating point loop ($x += dx$) into an integer loop ($i++$).
- ◆ Accumulation in double (*mixed precision*) or compensated algorithm.

Partial BasicBloc coverage

ddmin1 (integrate.hxx:16) random		nearest	
1	:②iostream(74)unitTest.cxx(65)	1	:②iostream(74)unitTest.cxx(65)
1	:②unitTest.cxx(65)iostream(74)	1	:②unitTest.cxx(65)iostream(74)
111168	:②cmath(185)	110988	:②cmath(185)
6	:②integrate.hxx(15)cmath(185)	6	:②integrate.hxx(15)cmath(185)
	integrate.hxx(11,13-15) F?		integrate.hxx(11,13-15) F?
	ostream(196) F		ostream(196) F
	ios_base.h(84,88,731)integrate.hxx(11,13-15) F?		ios_base.h(84,88,731)integrate.hxx(11,13-15) F?
≈26 lines skipped ≈			

partial (last iter) ddmin1 (integrate.hxx.16) random		partial (last iter) nearest	
100062	:②cmath(185)	99883	:②cmath(185)
1	:②integrate.hxx(15)cmath(185)	1	:②integrate.hxx(15)cmath(185)
100063	:②integrate.hxx(15-17) F?	99884	:②integrate.hxx(15-17) F?
1	:②iomanip(240)stl_algobase.h(237) ↔ integrate.hxx(11,13-15) F? ostream(196) F	1	:②iomanip(240)stl_algobase.h(237) ↔ integrate.hxx(11,13-15) F? ostream(196) F
≈11 lines skipped ≈			

How to specify coverage point

- ◆ client request : VERROU_DUMP_COVER
- ◆ IOMatch script

```

cmatch:      100000 * *
apply: dump_cover
cmatch:      10000 * *
apply: dump_cover

```

Temporal Delta-debug

- ◆ verrou_dd_task (with manual definition in source or automatic for python line)
- ◆ verrou_dd_stdout (task automatically detected thanks to IOMatch)

```
1 verrou_dd_stdout --nruns=5 ddRun.sh ddCmp.py
```

```
__verrou__stdout__init__
```

```
1 * *  
10 * *  
100 * *  
1000 * *  
10000 * *  
100000 * *
```

(a) Search space.

ddmin	match line
ddmin0	10000 * *

(b) Result.

- ◆ Warning : pay attention to bufferisation.
- ◆ It is possible to modify stdout (and so task) thanks a user script.

Mixed precision search for inner/outer iteration

```
CMD="etest2 100 100 1 res.dat rh.dat  
      -e ii -i bicgstab -p ilu ←  
      -ilu_fill 2  
      -print out -eprint out"  
valgrind --tool=verrou  
      --float=yes  
      --exclude=exclude.ex  
$CMD > $1/res.out
```

```
> cat exclude.ex  
lis_vector_dot *
```

```
...  
iteration: 47 relative residual = 2.102138E-10  
iteration: 48 relative residual = 2.340582E-11  
iteration: 49 relative residual = 7.864099E-12  
iteration: 50 relative residual = 4.231160E-12  
iteration: 51 relative residual = 2.719284E-12  
iteration: 52 relative residual = 8.079880E-13  
linear solver status : normal end  
  
iteration: 2 relative residual = 7.941629E-02  
initial vector x : all components set to 0  
precision : double  
linear solver : BiCGSTAB  
preconditioner : ILU(2)  
convergence condition : ||b-Ax||_2 <= 1.0e-12 * ||b-Ax_0||_2  
matrix storage format : CSR  
iteration: 1 relative residual = 9.351250E-02  
iteration: 2 relative residual = 6.529751E-02  
iteration: 3 relative residual = 2.403754E-02  
iteration: 4 relative residual = 1.016325E-02  
...  
Inverse: mode number = 0  
Inverse: eigenvalue = 1.934862e-03  
Inverse: number of iterations = 1000  
Inverse: elapsed time = 3.840000e+02 sec.  
Inverse: preconditioner = 1.280000e+02 sec.  
Inverse: matrix creation = 0.000000e+00 sec.  
Inverse: linear solver = 2.560000e+02 sec.  
Inverse: relative residual = 3.699645e-06
```

search space	ddmin subset	
	all	all \{\dot\}
outer-0 matrix storage format : CSR	ddmin15	OK
preconditioner : ILU	OK	OK
initial vector x : all components set to 0	OK	OK
outer-0 iteration: 10 relative residual = *	ddmin15	OK
outer-0 iteration: 20 relative residual = *	OK	OK
outer-0 iteration: 30 relative residual = *	OK	OK
outer-0 iteration: 40 relative residual = *	OK	OK
outer-0 linear solver status : normal end	OK	OK
outer iteration: 0 relative residual = *	OK	OK
outer-5 matrix storage format : CSR	ddmin0	ddmin0
outer-5 iteration: 10 relative residual = *	ddmin1	ddmin1
outer-5 iteration: 20 relative residual = *	ddmin16	ddmin2
outer-5 iteration: 30 relative residual = *	ddmin14	OK
outer-5 iteration: 40 relative residual = *	ddmin16	OK
outer-5 linear solver status : normal end	OK	OK
outer iteration: 5 relative residual = *	OK	OK
outer-10 matrix storage format : CSR	ddmin2	ddmin3
outer-10 iteration: 10 relative residual = *	ddmin3	ddmin4
outer-10 iteration: 20 relative residual = *	ddmin4	ddmin5
outer-10 iteration: 30 relative residual = *	ddmin5	ddmin6
outer-10 iteration: 40 relative residual = *	OK	OK
outer-10 linear solver status : normal end	OK	OK
outer iteration: 10 relative residual = *	ddmin6	ddmin7
outer-15 matrix storage format : CSR	ddmin7	ddmin8
outer-15 iteration: 10 relative residual = *	ddmin8	ddmin9
outer-15 iteration: 20 relative residual = *	ddmin9	ddmin10
outer-15 iteration: 30 relative residual = *	ddmin10	ddmin11
outer-15 linear solver status : normal end	ddmin11	ddmin12
outer iteration: 15 relative residual = *	ddmin12	ddmin13
outer-15 iteration: 40 relative residual = *	ddmin13	ddmin14

Interflop verification tools

Verrou is not the silver bullet tool. As a user I'm interested in other Interflop verification tools :

- ◆ **Verificarlo** <https://github.com/verificarlo/verificarlo>
 - ▶ Computation time (especially with share memory)
 - ▶ Better Debugger compatibility
- ◆ **Pene** <https://github.com/aneoconsulting/PENE>
 - ▶ Windows Portability
 - ▶ avx512
- ◆ **CADNA** <https://cadna.lip6.fr/>
 - ▶ Algorithm which take into account accuracy estimation
 - ▶ Useful to perform synchronous/asynchronous comparison.
- ◆ **FLDLib** <https://github.com/fvedrine/fldlib>
 - ▶ High level of confidence for tricky kernel.

Conclusions and perspectives

- ◆ **VERROU** provides an easy floating point error estimation ;
- ◆ **VERROU** provides error localization ;
- ◆ **VERROU** is used with industrial code ;
- ◆ **VERROU** is open-source and available:
<https://github.com/edf-hpc/verrou>

Perspectives

- ◆ Performance improvements (instrumentation, vectorization, delta-debug parallelism ...)
- ◆ Localization improvements (delta-debug search space, amplification detection ...)

Question?

Performance

type compilation option	double		float	
	O0	O3	O0	O3
tool_none	x6.3	x6.4	x7.0	x8.4
nearest	x10.0	x22.3	x10.6	x27.9
nearest-nc	x10.1	x21.8	x10.4	x27.2
random	x15.4	x41.2	x17.7	x54.1
average	x17.4	x47.5	x21.0	x66.0
random_det	x16.6	x46.0	x19.8	x64.7
random_comdet	x16.8	x47.2	x20.2	x66.8
random_scomdet	x19.4	x56.2	x23.4	x79.8
average_det	x19.2	x53.1	x23.5	x77.5
average_comdet	x20.2	x56.5	x24.9	x85.3
average_scomdet	x20.0	x56.7	x25.3	x85.4
sr_monotonic	x20.4	x57.4	x25.0	x81.5
sr_smonotonic	x20.5	x57.0	x25.3	x83.1

Number of samples

Confidence level $1 - \alpha$	Probability p								
	0.66	0.75	0.8	0.85	0.9	0.95	0.99	0.995	0.999
0.66	3	4	5	7	11	22	108	216	1079
0.75	4	5	7	9	14	28	138	277	1386
0.8	4	6	8	10	16	32	161	322	1609
0.85	5	7	9	12	19	37	189	379	1897
0.9	6	9	11	15	22	45	230	460	2302
0.95	8	11	14	19	29	59	299	598	2995
0.99	12	17	21	29	44	90	459	919	4603
0.995	13	19	24	33	51	104	528	1058	5296
0.999	17	25	31	43	66	135	688	1379	6905

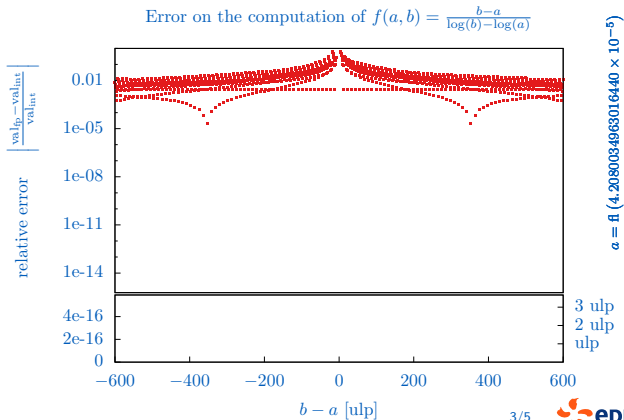
Confidence Intervals for Stochastic Arithmetic, Devan Sohier, Pablo De Oliveira Castro, François Févotte, Bruno Lathuilière, Eric Petit, Olivier Jamond

Bug fix example (1/3)

$$f(a, b) = \begin{cases} a & \text{if } a = b \\ \frac{b-a}{\log(b)-\log(a)} & \text{if not} \end{cases}$$

Empirical study

- outside the code
- around the problematic
- reference = interval arithmetic

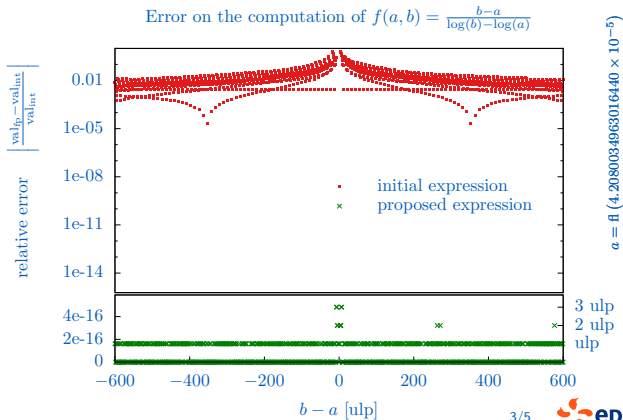


Bug fix example (1/3)

$$f(a, b) = \begin{cases} a & \text{if } a = b \\ \frac{b-a}{\log(b)-\log(a)} & \text{if not} \end{cases} \xrightarrow[\text{manual}]{\text{rewriting}} f(a, b) = \begin{cases} a & \text{if } a = b \\ a \frac{\frac{b}{a}-1}{\log(\frac{b}{a})} & \text{if not} \end{cases}$$

Empirical study

- outside the code
- around the problematic
- reference = interval arithmetic



Bug fix example (1/3)

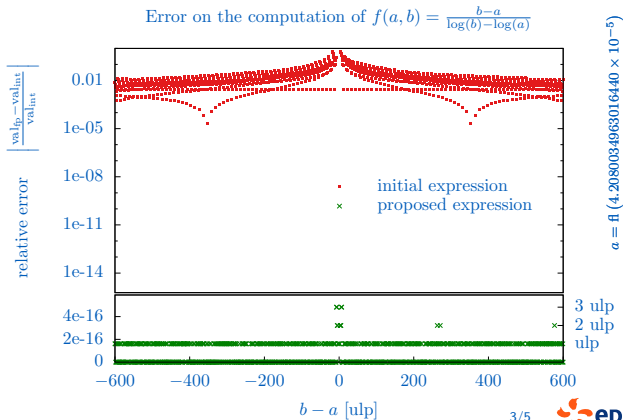
$$f(a, b) = \begin{cases} a & \text{if } a = b \\ \frac{b-a}{\log(b)-\log(a)} & \text{if not} \end{cases} \xrightarrow[\text{manual}]{\text{rewriting}} f(a, b) = \begin{cases} a & \text{if } a = b \\ a \frac{\frac{b}{a}-1}{\log(\frac{b}{a})} & \text{if not} \end{cases}$$

Empirical study

- outside the code
- around the problematic
- reference = interval arithmetic

Proof

- error bounded by 10 ulps

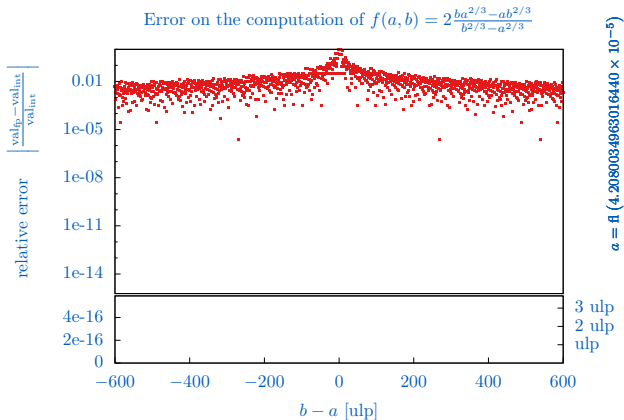


Bug fix example (2/3)

$$f(a, b) = \begin{cases} a & \text{if } a = b \\ 2 \frac{ba^{2/3} - ab^{2/3}}{b^{2/3} - a^{2/3}} & \text{if not} \end{cases}$$

Empirical study

- outside the code
- around the problematic
- reference = interval arithmetic



Bug fix example (2/3)

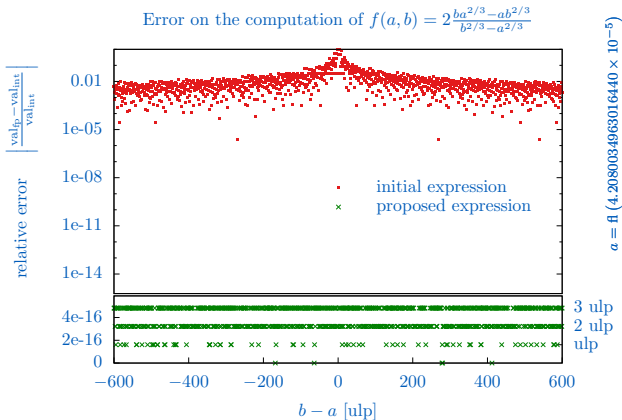
$$f(a, b) = \begin{cases} a & \text{if } a = b \\ 2 \frac{ba^{2/3} - ab^{2/3}}{b^{2/3} - a^{2/3}} & \text{if not} \end{cases}$$

wolfram
alpha

$$f(a, b) = 2 \frac{a^{2/3} b^{2/3}}{a^{1/3} + b^{1/3}}$$

Empirical study

- outside the code
- around the problematic
- reference = interval arithmetic



Bug fix example (2/3)

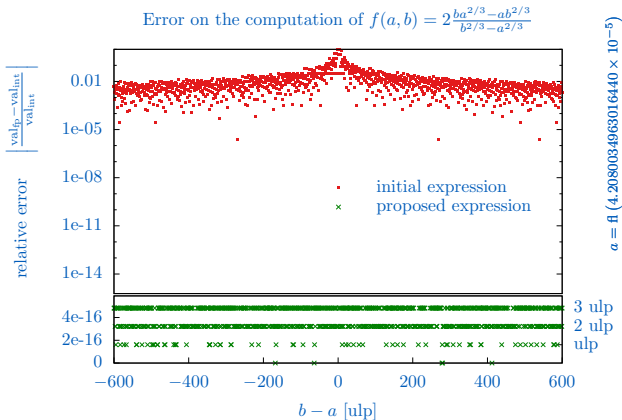
$$f(a, b) = \begin{cases} a & \text{if } a = b \\ 2 \frac{ba^{2/3} - ab^{2/3}}{b^{2/3} - a^{2/3}} & \text{if not} \end{cases}$$

wolfram
alpha

$$f(a, b) = 2 \frac{a^{2/3} b^{2/3}}{a^{1/3} + b^{1/3}}$$

Empirical study

- outside the code
- around the problematic
- reference = interval arithmetic



Bug fix example (3/3)

$$f_n(a, b) = \begin{cases} a & \text{if } a = b \\ (n-1) \frac{b^{\frac{1}{n}} - a^{\frac{1}{n}}}{a^{\frac{1}{n}-1} - b^{\frac{1}{n}-1}} & \text{if not} \end{cases} \xrightarrow[\text{rewriting}]{\text{manual}} f_n(a, b) = \frac{n-1}{\sum_{i=1}^{n-1} a^{\frac{i-n}{n}} b^{\frac{-i}{n}}}$$

Error on the computation of $f_n(a, b) = \frac{(n-1)(b^{1/n} - a^{1/n})}{a^{1/n-1} - b^{1/n-1}}$ with $n = 7$

