Neural Network Precision Tuning Using Stochastic Arithmetic

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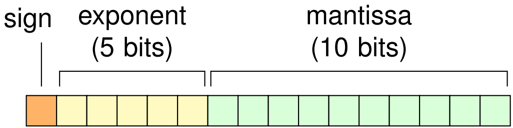
Introduction

IEEE754 Standard types

Format	Name	Length	Sign	Mantissa Length	Exponent Length
binary16	Half	16 bits	1 bit	11 bits	5 bits
binary32	Single	32 bits	1 bit	24 bits	8 bits
binary64	Double	64 bits	1 bit	53 bits	11 bits



binary16 format

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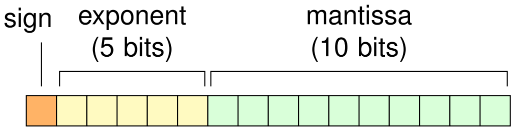


binary16 format

Reduced precision:

- Shorter execution time ☺
- Less volume of results exchanged (less memory used) ☺
- Less energy consumption ☺
- Less accurate results - rounding errors ☹

Related Work: Precision Auto-Tuning

Static Approach

- FPTuner [Chiang et al., 2017], SALSA [Damouche and Martel, 2018], TAFFO [Cherubin et al., 2020]

Dynamic Approach

- CRAFT HPC [Lam et al., 2013], Precimonious [Rubio-González et al., 2013], HiFPTuner [Guo et al., 2018] and PROMISE [Graillat et al., 2019]

On GPUs

- AMPT-GA [Kotipalli et al., 2019], GPUMixer [Ho et al., 2019], GRAM [Ho et al., 2021]

Related Work: Neural Network

Security and Robustness

[Singh et al., 2018], [Madry et al., 2019], [Lin et al., 2019], [Rakin et al., 2021], [Tjeng et al., 2019]

Impact of rounding errors

[Zombori et al., 2021]
Actual robustness $\neq$ Robustness given by a verifier

Related Work

Neural Network Precision Tuning [Ioualalen and Martel, 2019]

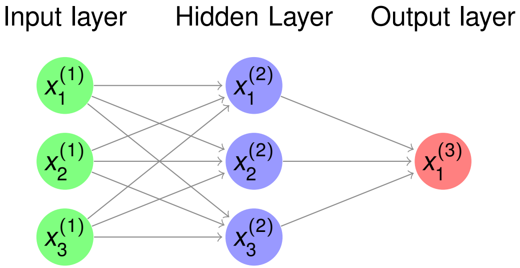
- floating-point auto-tuning algorithm that takes into account a given tolerance
- focus on interpolation networks, i.e. networks computing mathematical function
- precision optimized by solving a linear programming problem

Main differences with our work using PROMISE:

- focus on interpolation networks
- solving a linear programming problem $\neq$ Delta-Debug algorithm
- reference result altered by rounding errors

Neural Network

An artificial neural network is a computing system defined by several neurons distributed on different layers



$$x^{(k+1)} = g^{(k+1)}(W^{(k+1)}x^{(k)} + b^{(k+1)})$$

with W the weight matrix, b the bias vector and g the activation function

Neural Network

The activation function is a non-linear and often monotonous function.

- Sigmoid: $f(x) = \frac{1}{1+e^{-x}}, \forall x \in \mathbb{R}$
- Hyperbolic Tangent ($\texttt{tanh}$)
- Rectified Linear Unit (ReLU): $max(0, x)$
- Softmax: normalizes input vector $x \in \mathbb{R}^n$ into a probability distribution in σ
  $\sigma(x)_j = \frac{e^{x_j}}{\sum_{n=1}^{n} e^{x_i}}, \forall j = 1...n$

Discrete Stochastic Arithmetic (DSA) [Vignes 2004]

Classic arithmetic

$\boxed{A \oplus B \rightarrow R}$

$R = 3.14237654356891$

DSA

Random rounding

$A_1 \oplus B_1 \longrightarrow R_1$

$A_2 \oplus B_2 \longrightarrow R_2$

$A_3 \oplus B_3 \longrightarrow R_3$

$R_1 = \mathbf{3.14}1354786390989$
$R_2 = \mathbf{3.14}3689456834534$
$R_3 = \mathbf{3.14}2579087356598$

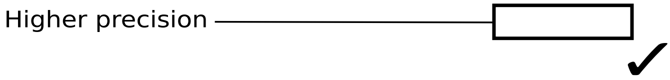
Rounding error analysis based on the CESTAC method:

- Allows the estimation of round-off error propagation
- Based on random rounding mode
- Computing N samples of a result R, value of R becomes $\bar{R}$
  In practice $N = 3$
- Number of correct digits thanks to Student's test with confidence level 95%

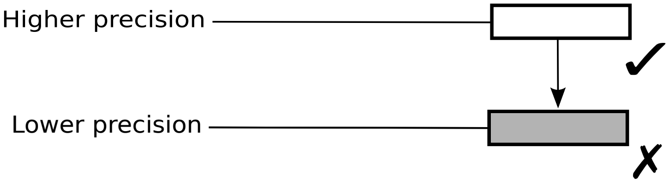
- implements stochastic arithmetic for C/C++ or Fortran codes
- provides stochastic types: 3 values of a variable + 1 integer being the accuracy
- returns value with the exact number of correct digits

PROMISE promise.lip6.fr ([Graillat et al., 2019], [Jézéquel et al., 2021])

- Provides a mixed-precision code taking in account a required accuracy
- Uses CADNA to validate a configuration
- Uses the Delta-Debug algorithm to test the different configurations, not exhaustive but mean complexity in $O(nlog(n))$ for n variables [Zeller, 2019]


Higher precision
✓

Searching for a valid configuration with 2 types



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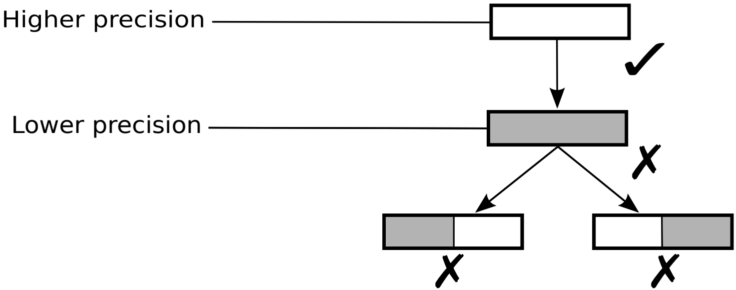

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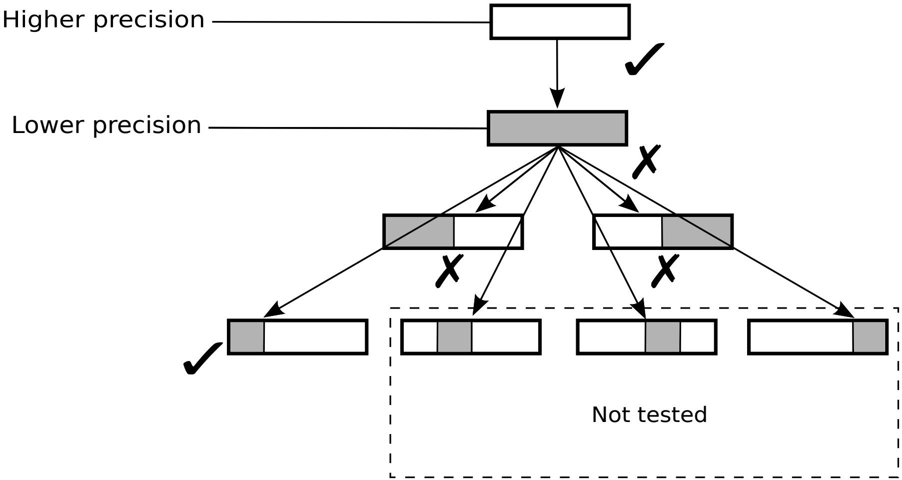
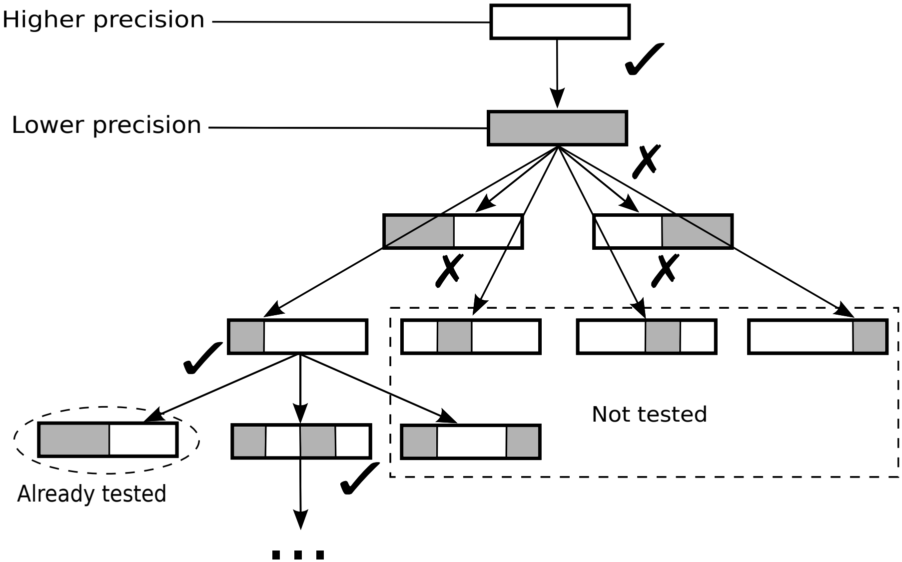
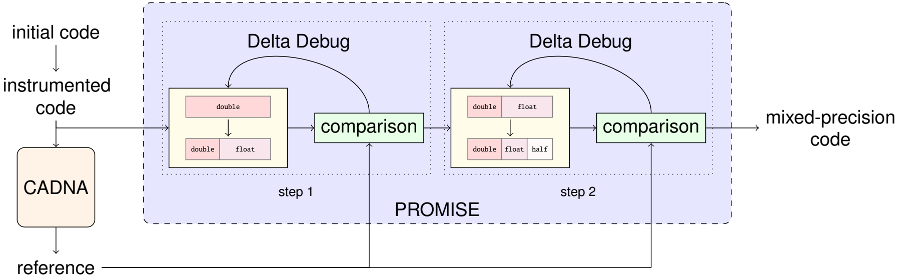


Searching for a valid configuration with 2 types



PROMISE

instrumented code = code with PROMISE variables, custom types variables that PROMISE recognizes and will consider tweaking



- step 1: lower from double to single precision
- step 2: lower from single to half precision

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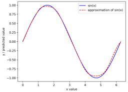
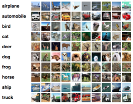
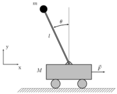
Neural networks studied

4 different Neural Networks:

- Sine NN: approximation of sine function
- MNIST NN: classification of handwritten digits (MNIST Database)
- CIFAR NN: classification of pictures among 10 classes (dogs, cats, deer, car, boat...) (CIFAR10 Database)
- Inverted Pendulum: computation of a Lyapunov function [Chang et al., 2020]



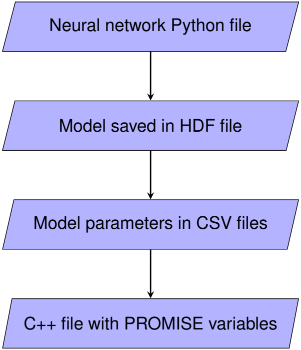




Methodology

- Neural Networks created and trained in Python code with Keras or PyTorch
- Python scripts to pass them into C++ instrumented code

- One type per neuron
- One type per layer
  With input as double precision.



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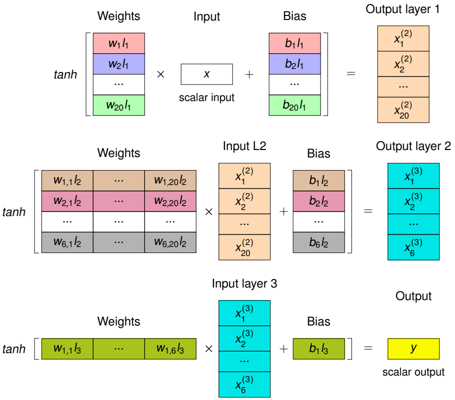
Sine NN

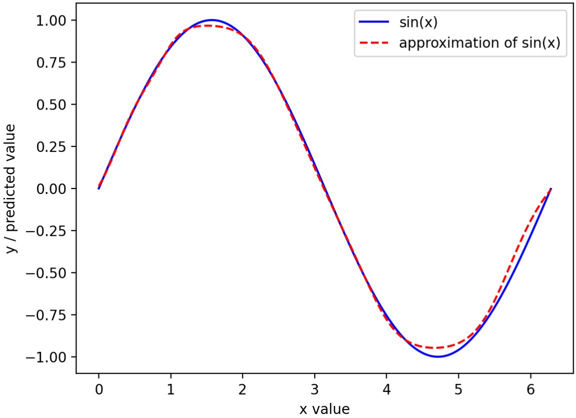
Approximation of sine function:

- Scalar input
- 3 dense layers with tanh activation function:
  - 20 neurons → 21 types to set
  - 6 neurons → 7 types to set
  - 1 neuron → 2 types to set
- Scalar output

➔ 30 types to set in total

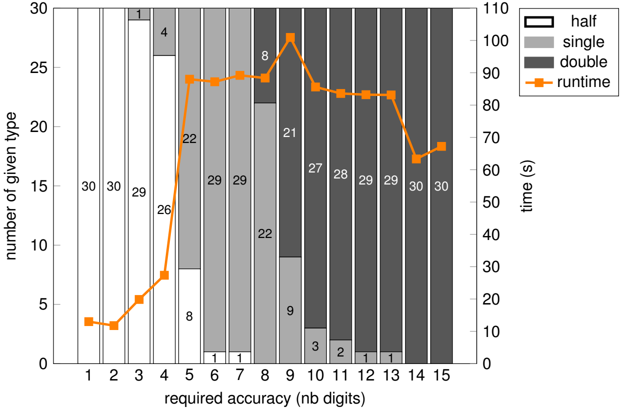


Sine NN

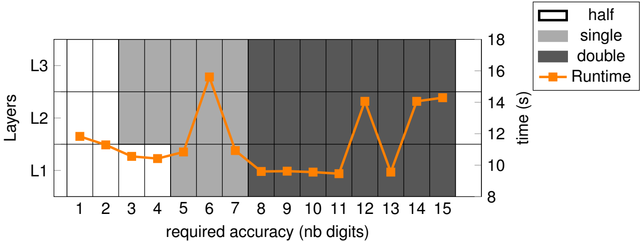


sin(x)
approximation of sin(x)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
y / predicted value
0
1
2
3
4
5
6
x value

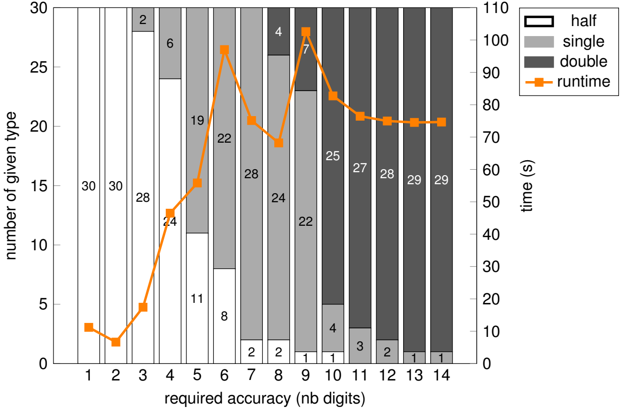
Sine NN w/ input=0.5



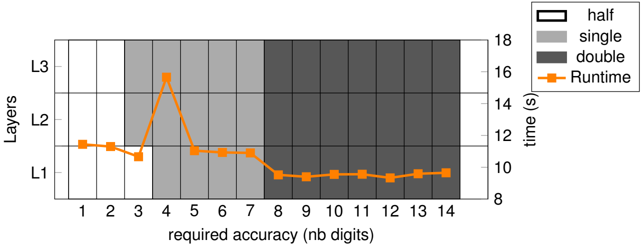
Sine NN w/ input=0.5



Sine NN w/ input=-2.37



Sine NN w/ input=-2.37



MNIST NN

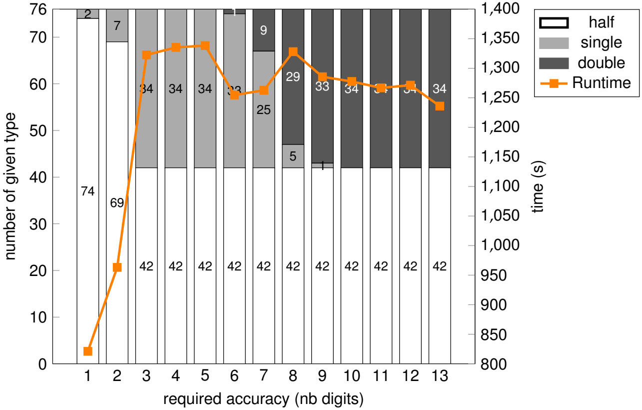
Classification of handwritten digits:

- Input: vector of size 784 (flatten image)
- 2 dense layers:
  - 64 neurons and ReLU activation function $\rightarrow$ 65 types to set
  - 10 neurons and softmax activation function $\rightarrow$ 11 types to set
- output vector of size 10: probability distribution for the 10 different classes

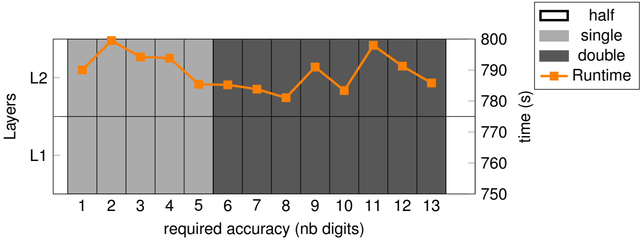
wikipedia.org

➔ 76 types to set in total

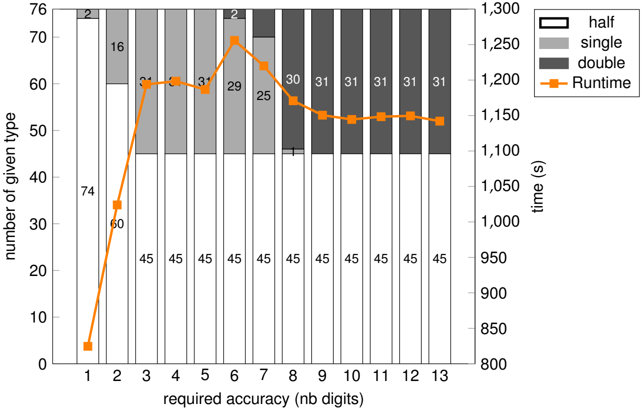
MNIST NN w/ input = test_data[61]



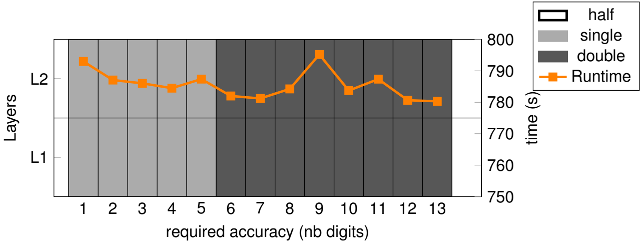
MNIST NN w/ input = test_data[61]



MNIST NN w/ input = test_data[91]



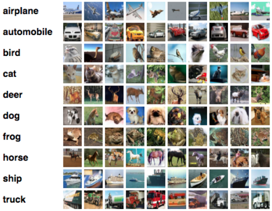
MNIST NN w/ input = test_data[91]



CIFAR NN

Classification of pictures in 10 classes:

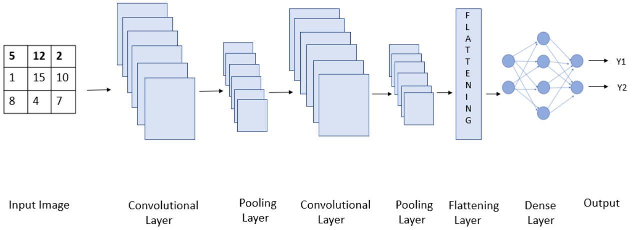
- Input: tensor of shape 32x32x3
- 8 layers:
  - Convolutional layer with 32 neurons and ReLU activation function → 33 types to set
  - Max-pooling layer of size (2x2) → 1 type to set
  - Convolutional layer with 64 neurons and ReLU activation function → 65 types to set
  - Max-pooling layer of size (2x2) → 1 type to set
  - Convolutional layer with 64 neurons and ReLU activation function → 65 types to set
  - Flatten layer → 1 type to set
  - Dense layer of 64 neurons and ReLU activation function → 65 types to set
  - Dense layer of 10 neurons and no activation function → 11 types to set
- output vector of size 10



cs.toronto.edu

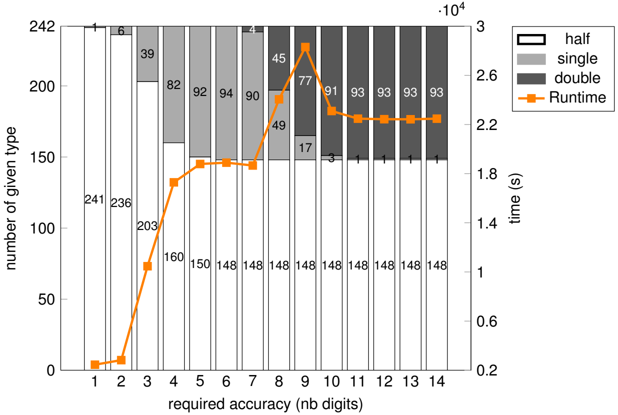
➔ 242 types to set in total

CIFAR NN

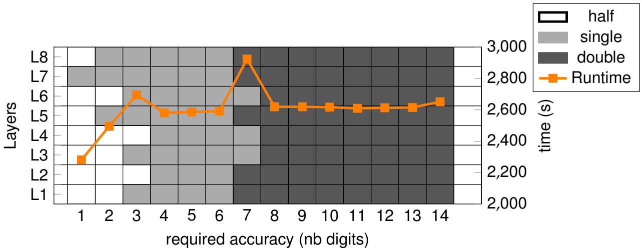


Aarya Brahmane - "Deep Learning with CIFAR-10" - https://towardsdatascience.com/

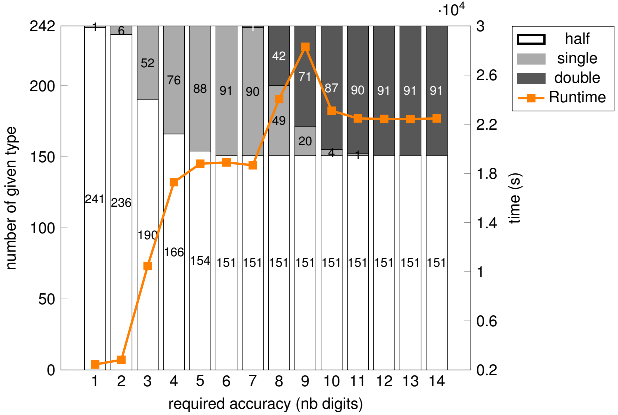
CIFAR NN w/ input=test_data[386]



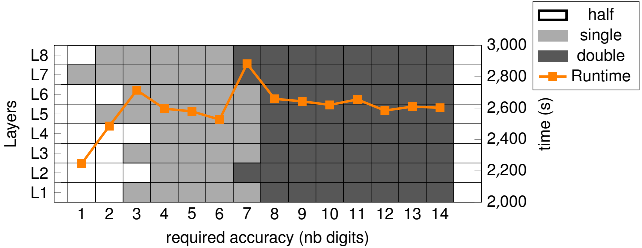
CIFAR NN w/ input=test_data[386]



CIFAR NN w/ input=test_data[731]



CIFAR NN w/ input=test_data[731]

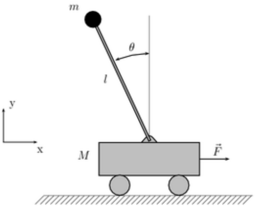


Pendulum NN

Learner to find a Lyapunov function:

- Input: state vector $x \in \mathbb{R}^2$
- 2 dense layers with tanh activation function:
  - 6 neurons $\rightarrow$ 7 types to set
  - 1 neuron $\rightarrow$ 2 types to set
- output vector of size 10

➔ 9 types to set in total



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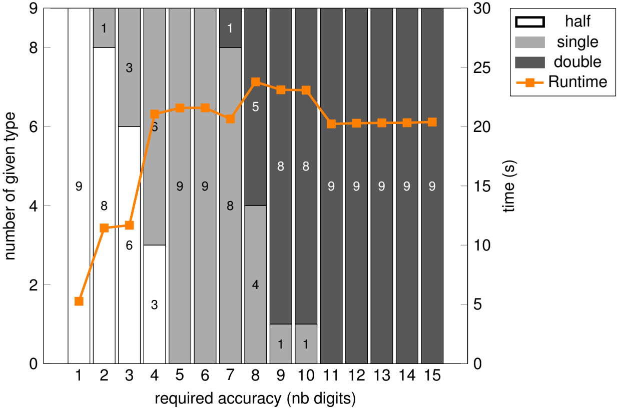
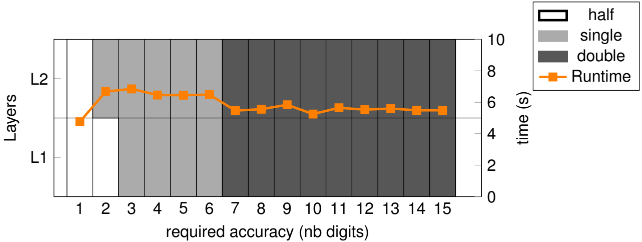
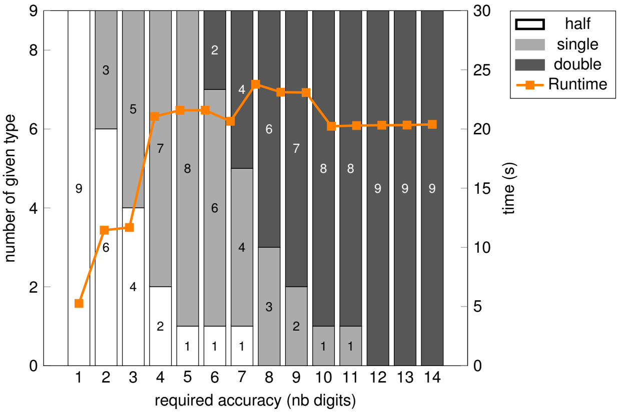
Inverted Pendulum w/ input=(0.5,0.5)



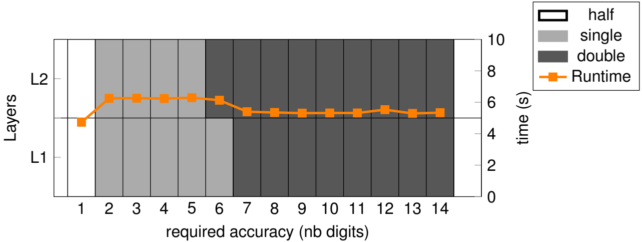
Pendulum NN w/ input=(0.5,0.5)



Inverted Pendulum w/ input=(-3,-6)



Pendulum NN w/ input=(-3,-6)



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Conclusion

- Can reduce precision per neuron using PROMISE
- No real advantage considering precision per layer except time cost and starting configuration for tuning per neuron

Future works

- Analyse on more inputs
- Analyse gain in time and memory
- Optimise the Delta-Debug algorithm [Hodován and Kiss, 2016]
- Consider the parallelization of PROMISE
- Extend PROMISE to GPUs and to arbitrary precision on FPGAs
- Use PROMISE with bfloat16

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